THE GENERIC MULTIVERSE VERSUS THE ELEMENTARY CLASS OF ZFC-MODELS

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Abstract. We want to compare the generic multiverse with the elementary class of first order models of (extensions or fragments of) ZFC. We first reframe the notion of generic multiverse by defining its elements as the "Tarski quotients" $V_\alpha^{\mathsf{B}}/_G$ of (rank initial segments) of V^{B} for B a complete boolean algebra of V and G a ultrafilter on B. We also analyze the notion of morphism between $\in$-models of (fragments of) ZFC and we argue that the correct notion of morphism between such models should preserve the Δ_0-formulae and the provably Δ_1-definable properties. Finally we combine these observations with work of Mansfield from the seventies to conclude that the models of set theory described by means of forcing sit inside the elementary class of models of (fragments) of the $\in$-theory of V as a universal subclass (e.g. every first order model of a suitable fragment of the $\in$- theory of V sits inside a model in the generic multiverse of the form $V_\alpha^{\mathsf{B}}/_G$). We also relate these results to model companionship properties of set theory. This is joint work with Giorgio Venturi and (to some extent) also with Francesco Parente and Tsvetlin Marinov.

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