

Meta-Pluralism: Moving Beyond the Universism/Multiversism Debate

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The debate between Universists and Multiversists in the foundations of set theory has already carried on for a while. It would seem that both Universists and Multiversists have good reasons to hold fast to their views, particularly insofar as each one may be articulated in a way which, potentially, makes the other redundant (cf. [Maddy, 2017] and [Hamkins, 2012]).

I have defended one of the two views, ie. Multiversism, myself (see, e.g., [Antos et al., 2015], [Friedman and Ternullo, 2016]) and claimed its compatibility with a naturalist standpoint ([Ternullo, 2019]).

However, in this paper, I point out an entirely different way in which the debate could (and should) advance by, first, raising basic *logical* issues which put pressure on characterisability of the debate itself, and then indicating how the practical stalemate between the two views could be overcome.

In particular, I will argue that when one equates the universe of sets V with models of set theory, one may be incurring a *categorical mistake*: V is not just a model among other models, but rather the embodiment of the whole of ‘set-theoretic reality’. So, on the one hand, the Universist cannot be characterised as someone who holds that there is a *single* universe, since the universe of sets is not to be confounded with models of set theory (provisionally taken to count as ‘universes’, too). On the other hand, the Multiversist cannot say that there are *many* universes of sets, as, in fact, by the reasoning above, there is just *one* (whilst, certainly, there are many models of the (first-order) axioms of set theory).

The tension may be resolved by granting both the universe of sets and models of set theory different, but mutually compatible, statuses, that is, roles within the practice of set theory itself. I call this view ‘meta-pluralism’,

insofar as it calls attention to equal legitimacy of ontological approaches viewed as alternative, but equally relevant, *methodological* approaches.

In particular, meta-pluralists will say that V expresses structural concerns about set theory, that is, a *determinate*, if incomplete, characterisation of the set-theoretic reality, along the lines of the picture and motivations laid out in [Isaacson, 2011]. Models of set theory, on the other hand, instantiate different ‘concepts of set’, ie., collections of incompatible set-theoretic properties, and, taken jointly within some set-theoretic *multiverse*, allow one to study the relationships among such different concepts, as imagined by [Hamkins, 2012].

My meta-pluralist may, thus, ultimately come to see the opposition between Universism and Multiversism as a *false dichotomy*, as both V and models of set theory play distinct roles in the foundations and practice of set theory, so, in particular, she will hold that that there is no fact of the matter about the ontological question of which of the two, V or the set-theoretic multiverse, represents the most correct picture of set-theoretic reality.

I will conclude the paper with some remarks on the consequences of meta-pluralism for set-theoretic (mathematical) *truth*.

References

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