Is the radical multiverse view coherent?

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Abstract

Hamkins has argued for a very radical version of set-theoretic multiversism, on which every model of first-order ZFC is on an ontological par with every other. Moreover, his responses to the categoricity arguments and attitude towards the natural number structure indicate that he is committed to a principle we call Bridging: Only those structures and statements that are agreed upon by all models of first-order ZFC are determinate. In this paper, following arguments of Barton and Koellner, we suggest that this view is not coherent since the notion of a model of first-order ZFC is itself not determinate within the ZFC-multiverse. We consider a response given by Barton—that the multiverse view should be regarded as proposing an algebraic framework for set-theoretic practice and argue that it is unworkable in the details in virtue of its dependence on another indeterminate notion, namely truth. We consider two responses available to the radical multiversist: (1.) Accept that the natural numbers are determinate, but otherwise hold on to the framework, or (2.) Plump for a further radicalisation of the view to only considering 'feasible' fragments of ZFC. Whilst both are coherent, we argue that each gives up a substantial part of the view. (1.) gives up Bridging, whereas (2.) gives up the naturalism that is often seen as a virtue of the position.